3-4 Derivative and Rates of Change

Learning Objectives:

I can calculate the position, velocity, acceleration, and speed for motion along a line.

I can calculate the position, velocity, acceleration, and speed for vertical motion.

I can interpret the position, velocity, acceleration, and speed using appropriate units.

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Motion Along a Line

Suppose that an object is moving along a coordinate line so that we know its position on that line as a function of time p = f(t) where t is in seconds and p is in meters.

The change in the object's position over time would be $\frac{dp}{dt}$. The rate of change of the object's position would be the object's Velocity and the units for $\frac{dp}{dt}$ would be m/sec.

$\frac{\Delta p}{\Delta t}$ $\frac{m}{sec}$

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??? Questions for Discussion ???

- What does the +/- on the velocity indicate?

- How would you find the speed of the object? What would be an expression for speed?

- What would be the meaning of $\frac{dv}{dt}$?

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Derivatives of a Position Function

p(t) = position of the object at time t

p'(t)= $\frac{dp}{dt}$ = the velocity of the object at time t

p''(t) = v'(t)= $\frac{dv}{dt} = \frac{d^2p}{dt^2}$ = the acceleration of the object at time t.

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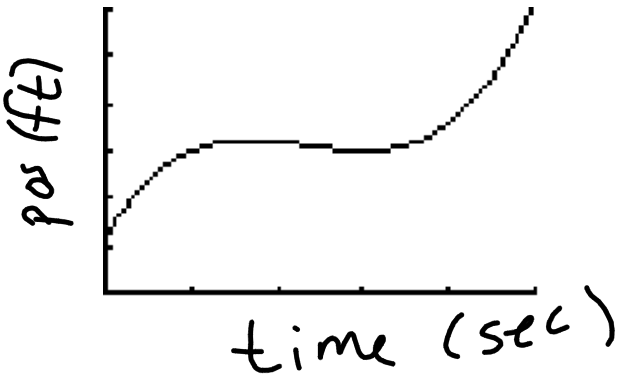
Ex1. An object's is moving back and forth on the number line and it's position is given by the equation $x(t) = t^3 - 7t^2 + 15t + 6$

Where x(t) is measured in ft and t is measured in sec

a.) Find the position of the object at time t = 1 sec, t = 2 sec t= 2.5 sec, and t = 5 sec.

b.) Graph x(t). $0 \le t \le 5$



$x(t) = t^3 - 7t^2 + 15t + 6$
Find position at:
t=1sec : 15 feet
t=2sec : 16 feet
t=2.5sec: 15.375 feet
t=5 sec: 31 feet

c.) What does the +/- on the position mean?

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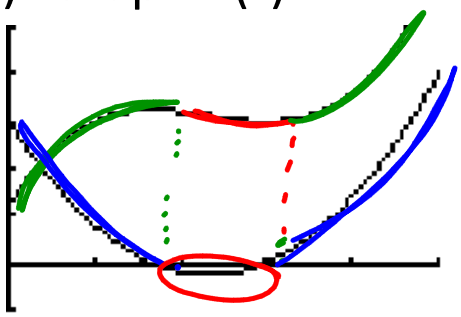
d.) Find the object's velocity as a function of time. $x(t) = t^3 - 7t^2 + 15t + 6$

$v(t) = 3t^2 - 14t + 15$

e.) Find the object's velocity at time t = 1 sec, t = 2 sec, t= 2.5 sec, and t = 5 sec.

Find velocity at:
t=2 sec: -1 ft/sec
t=2.5 sec: -1.25 ft/sec
t=5 sec: 20 ft/sec
$x''(t) = 6t - 14$

f.) Graph v(t)



g.) What does the +/- on the velocity mean?

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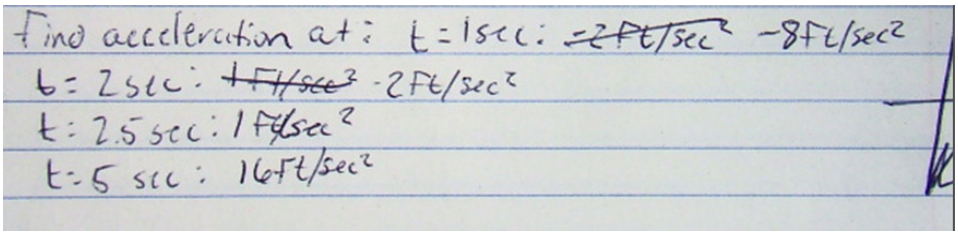
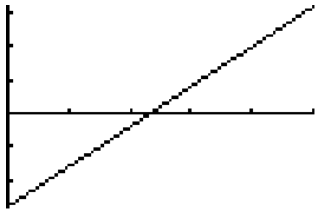
h.) Find the object's acceleration as a function of time. $x(t) = t^3 - 7t^2 + 15t + 6$

$v(t) = 3t^2 - 14t + 15$ $a(t) = 6x - 14$

i.) Find the object's acceleration at time t = 1 sec, t = 2 sec, t= 2.5 sec, and t = 5 sec.



j.) Graph a(t)



k.) What does the +/- on the acceleration mean?

l.) Completely describe the motion of the object at time t = 1 sec, t = 2 sec, t= 2.5 sec, and t = 5 sec.

m.) When is the object moving to the right? Moving to the left? Stopped?

n.) When is the object speeding up, slowing down, moving at a constant velocity?

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o.) When is the velocity of the object 7 ft/sec?

$x(t) = t^3 - 7t^2 + 15t + 6$

$v(t) = 3t^2 - 14t + 15$

$a(t) = 6t - 14$

$7 = 3t^2 - 14t + 15$

$0 = 3t^2 - 14t + 8$

$(3t - 2)(t - 4)$

$t = \frac{2}{3}$ sec, 4 sec

p.) When is the acceleration of the object 22 ft/sec^2?

$a = 6t - 14$

$22 = 6t - 14$

$36 = 6t$

$t = 6$ s

Object Moving Vertically under the Influence of Gravity

Free Fall

$$d(t) = -\frac{1}{2}gt^2$$

$$p(t) = -\frac{1}{2}gt^2 + h_0$$

t = time (seconds)
h_0 = Initial height
d(t) = displacement at time t
p(t) = position at time t

Gravity = 9.8 m/sec^2 or
32 ft/sec^2

Object Launched Vertically

$$d(t) = -\frac{1}{2}gt^2 + v_0t$$

$$p(t) = -\frac{1}{2}gt^2 + v_0t + h_0$$

t = time (seconds)
h_0 = initial height
v_0 = initial velocity
d(t) = displacement at time t
p(t) = position at time t

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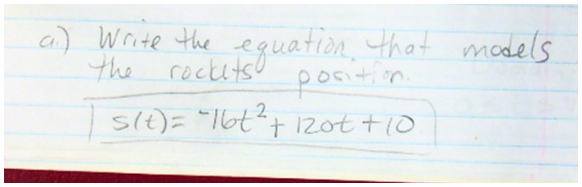
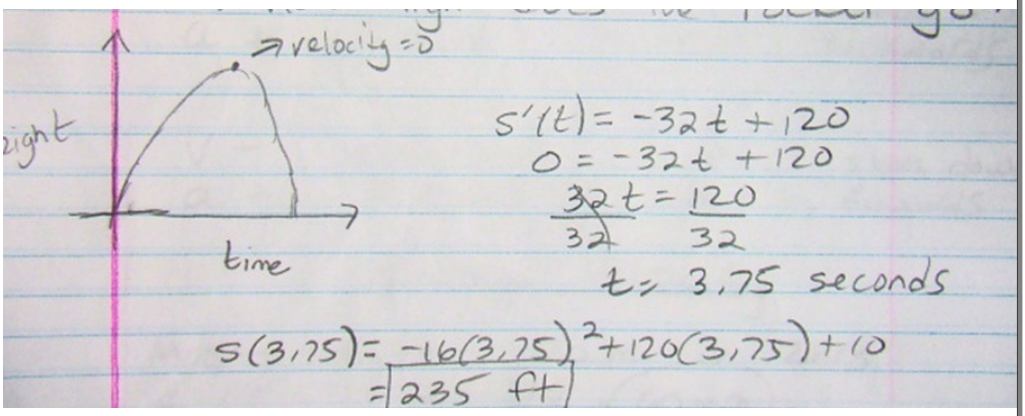
Ex2. A model rocket is shot vertically from 10 ft above the ground with an initial velocity of 120 ft/sec

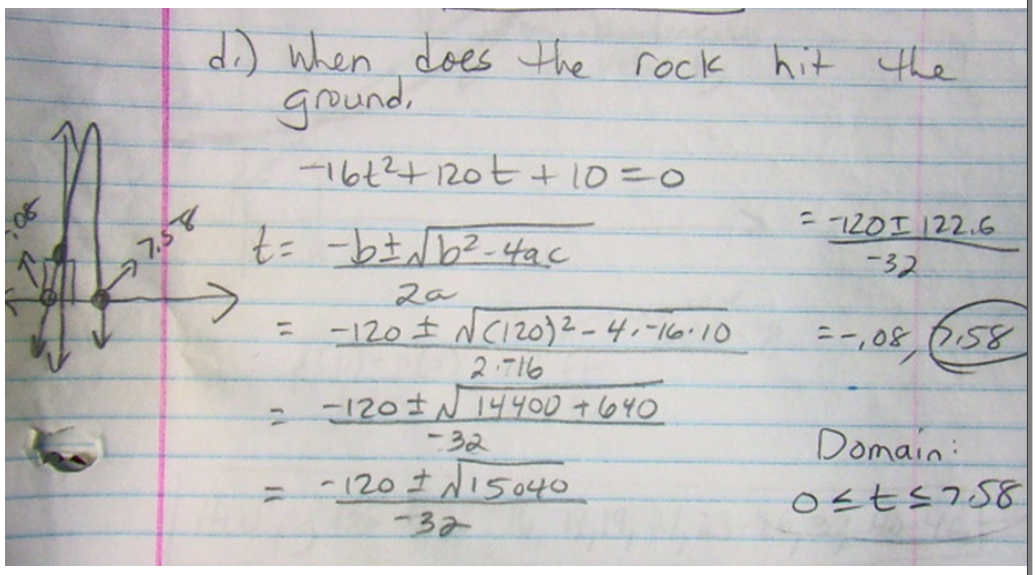
a.) Write the equation that models the rocket's position at time t.



b.) At what time does the rocket reach its maximum height? What is the maximum height?



c.) When does the rocket land on the ground?



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Average vs Instantaneous Velocity

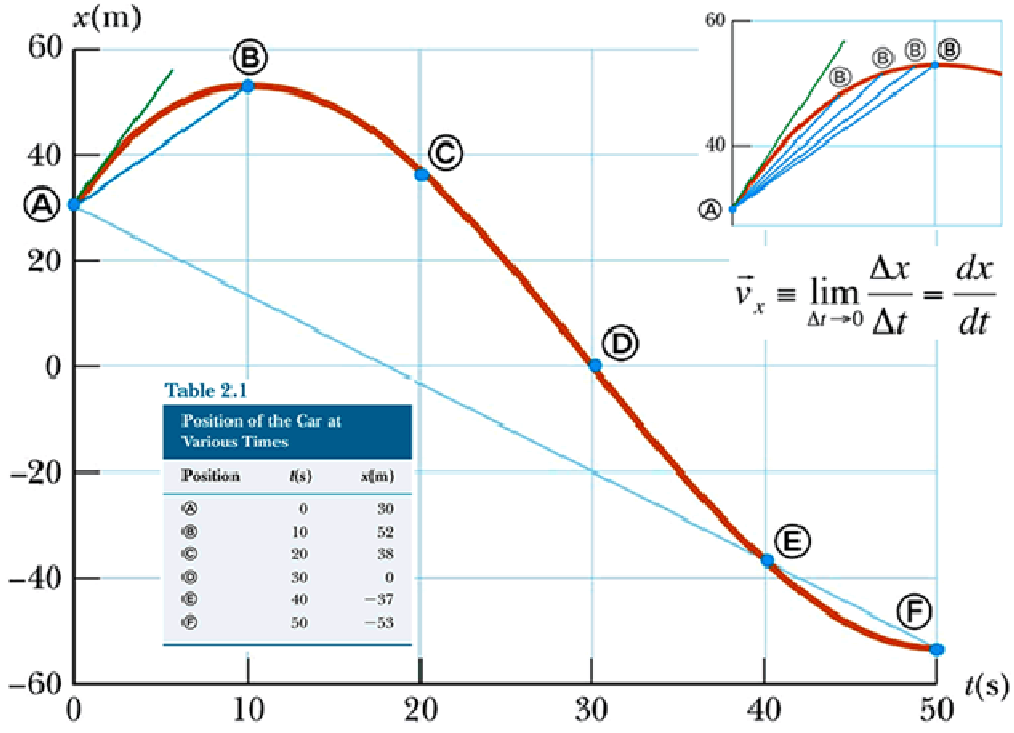

Table 2.1

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

$$\vec{v}_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

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Instantaneous Velocity

The derivative of the position function at time t=c.

position = f(t)

instantaneous velocity at time t = c would be f'(c)

Average Velocity

The slope of the secant line over the time interval from t = a to time t=b.

position = f(t)

position at time time t=a (a,f(a)) and the position at time t=b is (b,f(b))

average velocity from time t=a to time t=b would be

$$\frac{f(b)-f(a)}{b-a}$$

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Homework

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